ORIGINAL ARTICLE

A Preliminary Experimental Verification On the Possibility of Bell Inequality Violation in Mental States

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Abstract

For the first time we perform an experiment to test the possibility of Bell's inequality violation in mental states, during perception-cognition in humans**.** We study under a theoretical and experimental framework, a Bell-type test for human perception-cognition of ambiguous figures. It is performed a detailed analysis which demonstrates that, although we have not yet been able to violate Bell's inequality in the present performed experiment, there are strong theoretical arguments supporting our expectation to violate it by a simple articulation of the same experiment. In this framework we introduce for the first time what we retain to represent quantum cognitive observables. We consider that our analysis provides a solid ground for further investigations on quantum behaviour of cognitive systems. Therefore we reaffirm that quantum mechanics is a Giano Bifronte theory (two-faced Giano, a mythological God of the past), looking from one hand to physical reality and from the other hand to the sphere of mental reality and cognitive dynamics.

Key Words: perception-cognition of ambiguous figures, test of Bell inequality for mental states, cognition and quantum mechanics, quantum cognitive observables, neurophysiological correlates of quantum cognitive observables

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The Theoretical Framework

Bell's theorem was published in a fundamental paper of physics in 1964 (Bell, 1987). In 1975 Stapp defined Bell's theorem "*The most profound discovery of science*" (Stapp, 1994). We have to observe that he speaks here respect to science and not only respect to physics. Let us remember what the theorem states: it shows that the predictions of quantum mechanics are not intuitive and they relate the most fundamental issues of our physical, philosophical, epistemological and ontological reality. It emphasizes that no physical theory of local hidden variables may ever reproduces all of the predictions of quantum mechanics. It is the most famous legacy that we encounter in physics.

Of course, it is well known that Einstein was critical respect to standard interpretation of quantum mechanics. The celebrated EPR paper (Einstein et al., 1935) showed that the standard interpretation of such theory implies either its incompleteness or "spooky action-at-a-distance". Einstein wanted to get rid of the "action-at-a-distance" by assuming

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incompleteness of quantum mechanics and introducing "local hidden variables." Bell's theorem is considered to prove that it is possible to construct experiments in which it is impossible for any kind of interpretation based on "local hidden variables" to give the same predictions as quantum mechanics, providing a means of testing whether "action-at-a-distance" actually occurs. Many experiments have been conducted in physics confirming fully the validity of quantum theory (Aspect, 1999).

Rather recently, on the theoretical plane, analysis has been performed on probabilistic assumptions of Bell's formulation. One of us, Andrei Khrennikov (Khrennikov, 2004), has emphasized that J. Bell wrote about probability without to specify the concrete axiomatic of probability theory. His analysis shows that Bell did not apply the classical probability model, which, as it is well known, is based on Kolmogorov model to describe classical physical framework. In substance, he introduced his own probabilistic model and compared it with quantum mechanics. The crucial point for the present paper is that he did not pay attention to conditional probabilities. In detail, (Khrennikov, 2004; Wigner, 1970) in this model it was shown that the conditional probabilities in Bell elaboration cannot be defined by classical Bayes formula. In (Khrennikov, 2004; Wigner, 1970) it was used the approach based on Bell- type inequalities in the conventional approach of the Kolmogorov model. Khrennikov showed an analogous of Wigner inequality (Khrennikov, 2004; Wigner, 1970) for conditional probabilities, and, on this basis, evidenced in detail that the predictions of the conventional and quantum probability models must disagree.
This result represents an important advance because, in addition to the important theoretical elucidation that it reaches, on its basis we become finally able to perform experiments in various fields having in this manner the possibility to discuss the possible violation of Bell inequality in very different fields of interest. Our aim is confined to investigation of mental states brain dynamics, and in particular to perception and cognition of human beings. We will discuss in detail such features of the argument, but before let us add some further important considerations.

In this moment, but the question was in some manner signed from the starting of the theory, quantum mechanics suffers of a net dichotomy. We have a group of researchers who strongly are inclined to accept that quantum mechanics is a theory that speaks only about physical systems. In this manner, they exclude, as example, the possibility to investigate cognitive systems by this theory. On the contrary, there is another current of research that in principle is available to admit a larger field of pertinence of quantum mechanics, that is to say that it should not pertain uniquely to only physical systems but such enlarged field of reference could regard in particular brain dynamics, and, in particular, the sphere of the mental states in human beings. We retain to have given recently a net and definitive answer to this fundamental question. Accumulating results for about more than five years of experimental research, we have shown in a conclusive paper that mental state follow quantum mechanics during perception and cognition of ambiguous figures in a very large group of human subjects. The results have been published by us in *NeuroQuantology* and in arXiv (Conte 2008; Conte et al., 2008; Conte et al., 2003). We are firmly convinced that quantum theory is a theory of human cognition and recently (Conte et al., 2008) we have given a direct theoretical formulation on the manner in which wave function may be reconstructed for mental states and we have given direct results on the existence of quantum wave function and quantum interference effects in mental states through an experimental confirmation during perception and cognition in Humans. In these papers, we have also shown in detail that we think in a quantum mechanical manner through an experimental verification of existing quantum interference effects in the presumed cognitive anomaly that we call Conjunction Fallacy in psychology. The conclusion of such our last paper is that quantum mechanics is a theory of human cognitive dynamics. In order to illustrate such conclusion, according to Orlov (1996), we re-outline here that, though undescribed Nature certainly exists in the present formulations of science, scientific knowledge of Nature exists in the form of logically organized descriptions. When these descriptions become "too precise" in Orlov's words, as it is the case of quantum mechanics, the fundamental features of logic and language (and this is to say, of human cognition) acquire the same importance as the features of what is being described. At this level, it becomes impossible to separate the features of "matter per se" from the features of the logic and language (and thus of human cognition) used to describe it. All this happens in quantum mechanics and thus this is one of the qualitative reasons to admit a link that cannot be omitted between quantum mechanics and human cognition. The other more technical motivations are discussed in detail in the previous mentioned our papers.

In order to deepen our thesis, we would now perform a step on. It would be to analyse mental states of a group of human subjects verifying directly the possibility and the conditions under which a violation of Bell inequality may turn out in the analysis of mental states of subjects during their perception and cognition of ambiguous figures.

To this purpose in the present paper, we have performed the first experiment on Bell inequality, arriving to establish the conditions in which it may be violated. Further experiments are in progress in order to test definitively Bell's violation in perception-cognitive dynamics.

2. The Theoretical basis of the Experiment

We followed systematically the previous indications given (Khrennikov, 2004) to which the reader is sent for the relative deepening. Let us start considering the well-known Wigner inequality (Wigner, 1970) that this author evidenced for Bell inequality (see also Khrennikov, 2004).

Let $a,b,c=\pm 1$ be arbitrary dichotomous random variables on a single Kolmogorov space S. Then the following inequality holds true

$$\begin{aligned} &p(a=+1,b=+1)+p(c=+1,b=-1)\\ &\geq p(a=+1,c=+1) \end{aligned} \quad (\mathbf{1.1})$$

We recall now that conditional probabilities in the Kolmogorov model are defined by using Bayes' formula and thus writing

$$p(a=a_1/b=b_1)=\frac{p(a=a_1,b=b_1)}{p(b=b_1)} \quad (\mathbf{1.2})$$

The aim of the test that we perform is easily explained: A family of observables, which does not permit such statistical realistic description, will be called to follow a quantum like behaviour. We obtain that

$$\begin{aligned} &p(a=+1/b=+1)+p(c=+1/b=+1)\\ &\geq p(a=+1/c=+1) \end{aligned} \quad (\mathbf{1.3})$$

This is the inequality that we intend to explore in our experiments. Only observables with probabilities $\frac{1}{2}$ hold.

3. The Description of the Experiment

A group S of 144 subjects was divide into two subgroups, U of 72 subjects and V of 72 subjects. To the group U the test B was given with possible answers B^+ and B^-. To subject answering B^+, it was subsequently given the test A with possible answers A^+ and A^-, respectively. To subjects answering B^-, it was given instead the test with possible answers and C^-. To subjects pertaining the subgroup V, it was given instead the test C with C^+ and C^- as possible answers. To subjects answering C^+, it was finally given the test A with possible answers A^+ and A^-. The scheme of the experiment is given in Figure-1 and it follows rigorously the Eq-1.3.

Let us summarize the data and the results that were obtained

Investigation group of 144 subjects:

U = 72 subjects
V = 72 subjects
U_+^b = 37 subjects; V_+^c = 45 subjects
U_-^b = 35 subjects; V_-^c =27 subjects

$$\begin{aligned} &p(a=+/b=+)=\frac{22}{37}=0.5946\\ &p(c=+/b=-)=\frac{23}{35}=0.6571\\ &p(a=+/c=+)=\frac{24}{45}=0.5333 \end{aligned} \quad (\mathbf{1.4})$$

$$p(a=+\ /\ b=+) + p(c=+\ /\ b=-) \geq p(a=+\ /\ c=+) \quad (\mathbf{1.5})$$
$$0.5946 + 0.6571 \geq 0.5333$$

The result is that Bell inequality was not violated. Note, however, that in the hypothesis of the theorem shown in (Khrennikov, 2004), we must have that

$$p(b=+) = p(b=-) = \frac{1}{2};$$

$$p(c=+) = p(c=-) = \frac{1}{2}.$$

Instead we had asymmetrical results:

$p(b=+) = 0.514$

$p(b=-) = 0.486$

$p(c=+) = 0.625$

$p(c=-) = 0.375$

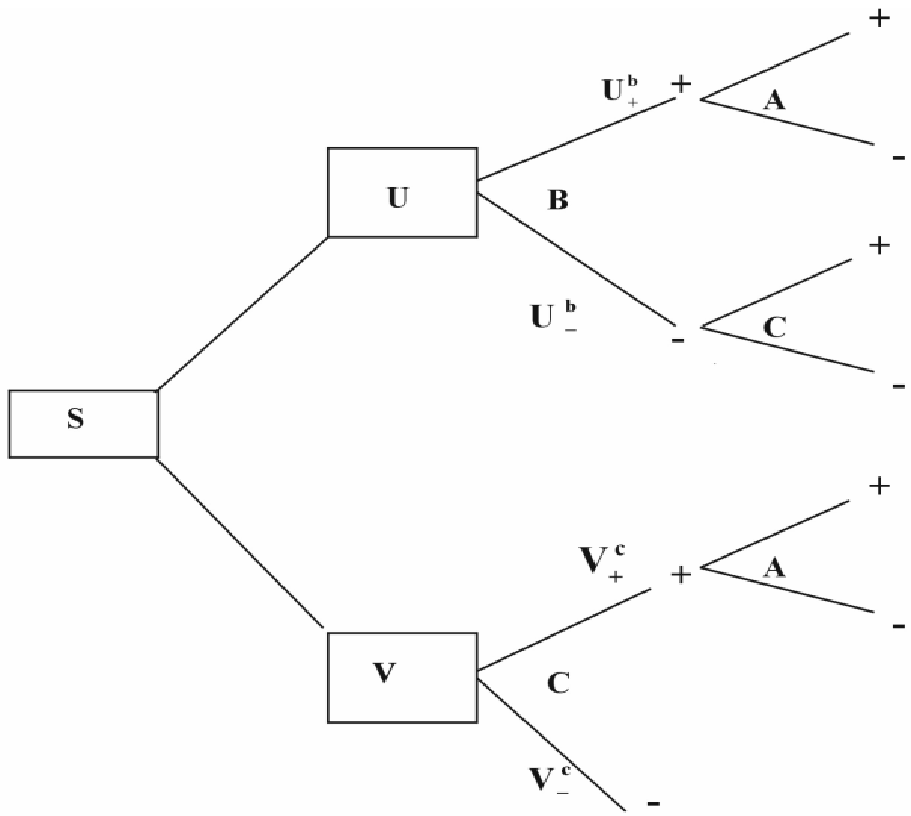
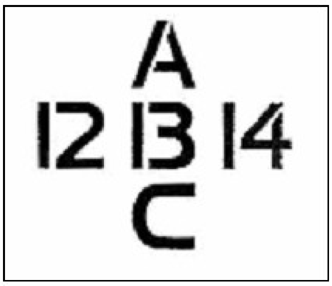
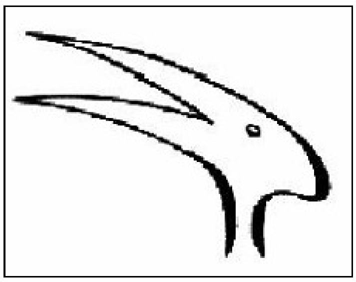
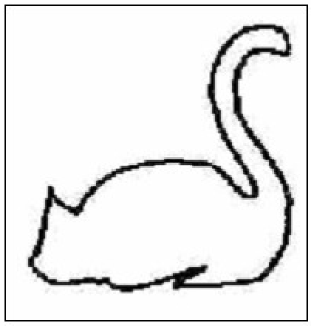


Figure 1. The scheme of the performed experiment.







Test B **Test C** **Test A**

We had to return to verification of old Wigner inequality, that is

$$p(a=+1,b=+1)+p(c=+1,b=-1)>=p(a=+1,c=+1).$$

Since in a Kolmogorov model we have

$$p(a=+ / b=+)=\frac{p(a=+\,;\,b=+)}{p(\,b=+)}$$

$$p(c=+ / b=-)=\frac{p(c=+\,;\,b=-)}{p(\,b=-)}$$

p(a=+ / c=+)= $\frac{p(a=+\,;\,c=+)}{p(\,c=+)}$

We calculated this time that

p(a=+,b=+)=p(a=+/b=+) p(b=+)
p(c=+, b=-)=p(c=+/b=-) p(b=-)
p(a=+, c=+)=p(a=+/c=+) p(c=+).

Consequently, we had

p(a=+;b=+)=0.5946x0.514 =0.30562
p(c=+,b=-)=0.6571x0.486=0.31935
p(a=+,c=+)=0.5333x0.625 =0.33331

That is to say
p(a=+1,b=+1)+p(c=+1,b=-1) $\geq$ p(a=+1,c=+1)
as 0.30562+0.31935 $\geq$ 0.33331. (**1.6**)

In conclusion, Bell inequality resulted not violated in such experimental stage of mental states behaviour during perception and cognition of ambiguous figures.

4. Refinement of the previous calculations.

We must give now some comments of decisive importance.

a. First of all we must outline that such results do not invalidate the results that recently we obtained in (Conte et al., 2008a and b) where we showed existing wave function for mental states and thus that mental states follow quantum mechanics and quantum interference effects during perception and cognition of ambiguous figures. Inequalities of Bell kind by themselves have nothing to do with quantum theory. As correctly outlined by J.H. Eberly (Eberly, 2002), contexts as downhill skiers (Meystre, 1984) and laundered socks (Bell, 1981) were used to demonstrate this. This is the first important point to outline.
b. In particular, we are in accord with Pitkänen (Pitkänen, 2008) who outlined that if the failure of inequality does not occur, this does not of course mean definitively that the system is classical but may be only that the quantum effects, in the proper version that we experienced, were not large enough.
c. According to our general view on quantum mechanics (Conte et al., 2006; 2007a; 2007b) and to Pitkänen (Pitkänen, 2008), Bell inequality that we have used in the present experimentation must be reformulated in some manner. We suggest that the questions we posed to human subjects, and consisting in Tests A, B, C previously seen, must be considered as perceptive-cognitive observables to be characterised by proper variables whose behaviour is in analogy to spins in spin pair of spin singlet states in an external magnetic field and determining a quantization axis. We must reformulate our experiment in order to have direct indications in this basic manner. We must account that we are in presence of cognitive variables and we must consider Bell's inequality in relation to such variables.
d. We arrive in this manner to the crucial conclusion of the present paper. As previously outlined and shown in detail in our previous papers above mentioned, quantum mechanics is a theory that has a profound link with human cognitive. We say that it represents a theory of human cognition. The mental observables are represented by the three abstract set (e_1, e_2, e_3) of the algebraic structure that we have used several times in our papers (see our previously quoted papers). They pertain to the sphere of cognitive dynamics, of mind activity and logic. In the physical sphere we have the case of physical particles where the projections of spins in three perpendicular reference directions are represented by the well known Pauli matrices,

$$S_1 = \frac{1}{2}he_1\,,\; S_2 = \frac{1}{2}he_2\,,\; S_3 = \frac{1}{2}he_3\,.$$

In addition, the projections of the magnetic moment are given by physical observables as $\mu e_1, \mu e_2, \mu e_3$. However, the $e_i\ (i=1,2,3)$, considered as algebraic abstract structure in the manner discussed by us in such our papers, represent mind-logical cognitive-observables. Orlov (1982), word for word, declared: *Every atomic proposition of classic logic can be represented by a diagonal operator –the third component of the Pauli algebra* e_3*. Therefore, as examined in our* *previous papers, our set (e_1, e_2, e_3) may be considered the basic algebraic set relating human cognitive variables.*

e. If in the present case of the experimentation, given in figure 1, we consider the tests A, B, C, as expressed by the algebraic sets (e_1, e_2, e_3), we arrive to find cases in which Bell inequality may be violated and we find the neurophysiological correlates to such experimental condition.

It remains now to show how Bell inequality could be violated in our experiment of Figure 1, based on mental states. Let us assume the Tests A, B, C, corresponding to mental observables, are given in the following manner:

Test A = $\cos\vartheta_1 e_3 + sen\vartheta_1 e_1$;

Test B= $\cos\vartheta_2 e_3 + sen\vartheta_2 e_1$;

Test C = $\cos\vartheta_3 e_3 + sen\vartheta_3 e_1$.

We have AB $\neq BA$, AC $\neq$ CA, BC $\neq$ CB
where
AB $\equiv$ Test A Test B = $\cos(\vartheta_1 - \vartheta_2) - isen(\vartheta_1 - \vartheta_2)e_2$
and
BA $\equiv$ Test B Test A= $\cos(\vartheta_1 - \vartheta_2) + isen(\vartheta_1 - \vartheta_2)e_2$
Still,
AC $\equiv$ TestATest C = $\cos(\vartheta_1 - \vartheta_3) - isen(\vartheta_1 - \vartheta_3)e_2$
and
CA $\equiv$ Test C Test = $\cos(\vartheta_1 - \vartheta_3) + isen(\vartheta_1 - \vartheta_3)e_2$ **(1.7)**
Finally,
BC $\equiv$ TestB Test C = $\cos(\vartheta_2 - \vartheta_3) - isen(\vartheta_2 - \vartheta_3)e_2$
and
CB $\equiv$ TestCTest B = $\cos(\vartheta_2 - \vartheta_3) + isen(\vartheta_2 - \vartheta_3)e_2$.

We know that

p(a= α_i / b= β_i)= $\left| < \phi_i^a, \phi_i^b > \right|^2$

being $\{\phi_i^a\}$ and $\{\phi_i^b\}$

normalized set of eigenvectors, respectively.

We have

$$\sigma(\vartheta)\phi_+(\vartheta) = \phi_+(\vartheta) \text{ with } \phi_+(\vartheta) = (\cos\frac{\vartheta}{2}, sen\frac{\vartheta}{2})$$

And

$$\sigma(\vartheta)\phi_-(\vartheta) = -\phi_-(\vartheta) \text{ with } \phi_-(\vartheta) = (-sen\frac{\vartheta}{2}, \cos\frac{\vartheta}{2})$$

In conclusion, instead of the (1.3) and the (1.5) we obtain now that

p(a=+ / b=+)= $\cos^2\frac{\vartheta_1 - \vartheta_2}{2}$

p(c=+ / b=-) = $sen^2\frac{\vartheta_3 - \vartheta_2}{2}$

and finally

p(a=+1,c=+1) = $\cos^2\frac{\vartheta_1 - \vartheta_3}{2}$ **(1.8)**

In conclusion, the (1.5), before non violated, now becomes

$$\cos^2\frac{\vartheta_1 - \vartheta_2}{2} + sen^2\frac{\vartheta_3 - \vartheta_2}{2} \geq \cos^2\frac{\vartheta_1 - \vartheta_3}{2} \quad \textbf{(1.9)}$$

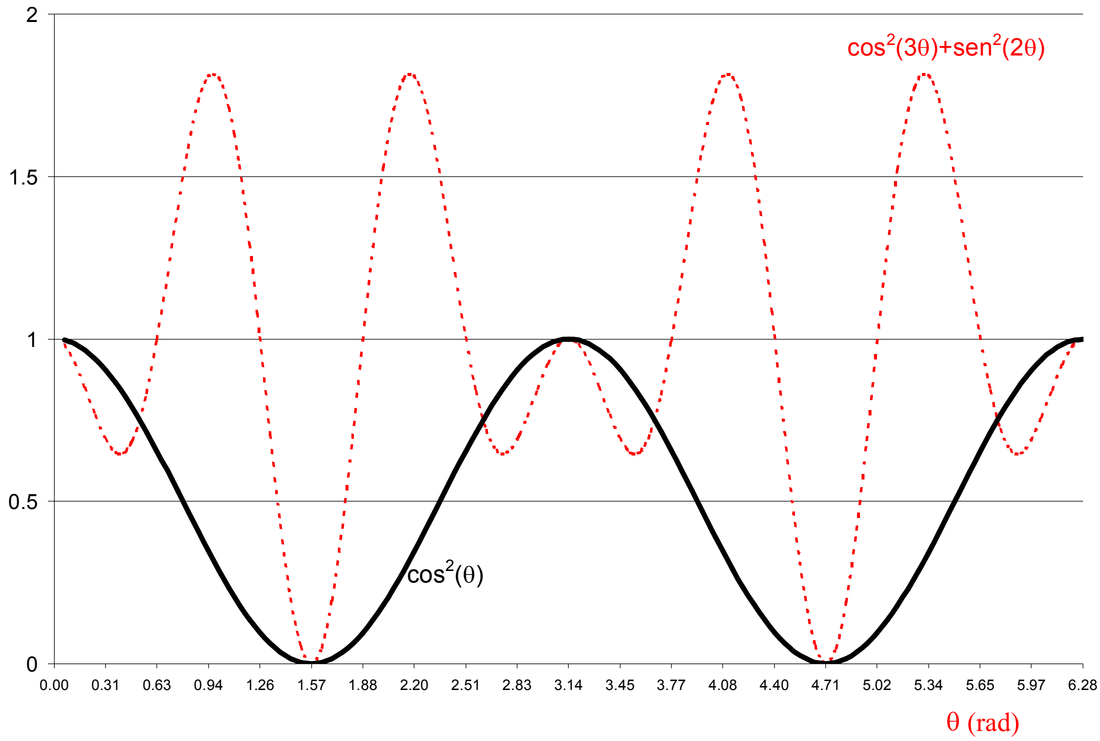
That represents the central equation of our paper. For details, see in particular ref (Khrennikov, 2004; Wigner, 1970). A simple solution of the (1.9) may be given assuming $\vartheta_1 = 0, \vartheta_2 = 6\vartheta, \vartheta_3 = 2\vartheta$. Instead of the (1.9), one obtains that

$$\cos^2 3\vartheta + sen^2 2\vartheta \geq \cos^2 \vartheta \quad \textbf{(1.10)}$$

In Figure-2 we give in red the sum of the two terms on the left and in blue the term on the right of the inequality (1.10). As expected, we see that we have regions of ϑ in which Bell's inequality for mental states may be violated.

The conclusion of such elaboration is that Bell's inequality may be violated also in the case of experiments based on mental states during perception and cognition of ambiguous figures.

Figure 2: Regions of Bell's violation (eqn. 1.10 in the text)



5. Neurophysiological Correlates

As seen, the (1.10) helps us in identifying the correct psychological and neurophysiological procedure we must follow during experimentation in order to identify Bell's inequality violation.

In order to perform our tests, we must account that perception and cognition in ambiguous figures are influenced by visual angle (Borsellino et al., 1981). Therefore, we used constant visual angle $V = 2arctg(S/2d) = 0.33\,rad.$ in our experimentation with S object's frontal linear size and *d* distance from the centre of the eyes for all the subjects. Let us represent in Figure 3 the neurophysiological condition.

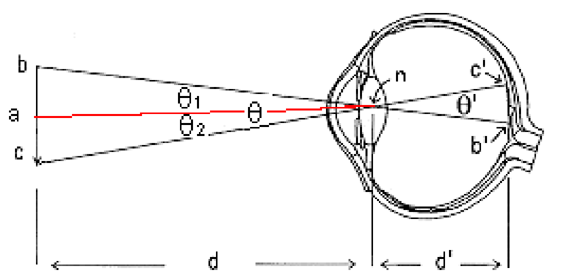


Figure 3. Visual angle and its neurophysiological correlate through the region of interested retina

We see that, among the possible stimulus variables affecting the reversal phenomenon, we have also the size of the ambiguous figure. The reason is very complicated from the viewpoint of the physiology but in brief, we may say that, when the size of the pattern changes, the visual angle subtended from the observer's eye varies and with it varies also the portion of retina involved. A net relation is established among visual angle, quantities describing perceptual alternation and neurophysiological correlates. Therefore, during investigation, we must realize figures each time with different side of the figure given as Test. Each time, during the Test, we will have a different portion of retina involved and with a selected rotation of the retina employed. It will represent the required relation between the neurophysiology and the spin-like quantization rotation relating cognitive observable that we outline in this paper as necessary to perform correct experimentation. Conditional probabilities will vary each time finally finding those giving Bell's inequality violation.

6. Conclusion

In the paper we studied (both theoretically and experimentally), a Bell-type test for human perception-cognition of ambiguous figures. We performed a detailed analysis that demonstrated that, although we have not yet been able to violate Bell's inequality in the present performed experiment, there are strong theoretical arguments supporting our expectation to violate it by a simple articulation of the same experiment. In this framework, we introduced for the first time what we retain to represent quantum cognitive observables. We consider that our analysis provides a solid ground for further investigations on quantum behaviour of cognitive systems. Therefore we reaffirm that quantum mechanics is a Giano Bifronte theory (two-faced Giano, a mythological God of the past), looking from one hand to physical reality and from the other hand to the sphere of mental reality and cognitive dynamics.

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